IV. Application to RAE 102-108% Airfoil

Having validated the procedure against NACA 0009 we applied the same to RAE 102-8% airfoil. This is a typical airfoil which gives a 'roof-top' pressure distribution. Again the comparison between given and obtained coordinates show (Table 2) good agreement. Table 1 gives σ for this airfoil also.

V. Conclusion

A new method for designing an airfoil by using "Wagner expansion" for an airfoil contour and an optimization technique for execution is given. This has the definite advantages compared to classical methods.

Firstly, it does not get into trouble of not closing at the trailing edge like other classical methods for, the expansion (1) itself ensures the closure at the trailing edge.

Secondly, it can be used for any general airfoil unlike the method of Lighthill.

Further improvements in the method of expansion for an asymmetrical airfoil and incorporation of Smith's method for a design C_L will enhance the usefulness of the method for designing an arbitrary airfoil at design C_L whose pressure distribution has been given. This is presently under the investigation by the authors.

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Optimal Trajectories for the Dive-Bombing Mission of a Fighter Aircraft

U.R. Prasad* and P.S. Subramanyam†

Indian Institute of Science, Bangalore, India

Introduction

THE dive-bombing mission of a fighter aircraft is inherently complex as it involves the interaction of the attacking pilot aircaft system and the enemy's target-defense system. The attacking pilot maneuvers his aircraft close to the target for better accuracy in aiming a weapon onto the target, and simultaneously tries to minimize his stay within the enemy's fire envelope for safe return. He has to arrive at a proper tradeoff between the twin objectives of safety and accuracy of target hit. The performance envelope of the aircraft and its handling qualities, the deteriorating effects of normal acceleration and psychological stress on the pilot, any operational limitations arising due to the topology of the target and the surrounding terrain, and the geometry of the target-defense system are some of the important aspects to be considered in this problem.

Recently pilot effects on weapon delivery accuracy in air-to-ground bombing have been studied as a linear stochastic problem.[1,2] However, the determination of optimal trajectories which bring out the best tradeoff between safety and accuracy of target hit, for varying stategies of the target defender is a problem which has not received much attention in the published literature, probably due to its complexity. In this Note we present some of the results obtained so far on this latter aspect.

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*Assistant Professor.

†Graduate Student. Presently Senior Scientific Officer, Control Division, Defense Research and Development Laboratory, Hyderabad, India.

Dive-Bombing Mission as a Trajectory Optimization Problem

The target-defense system normally consists of a set of radar-controlled anti-aircraft guns stragetically located around the target. The effective zone within which the shell from a gun has a high probability of hitting the attacking aircraft can be modeled as an ellipsoid whose axes represent the horizontal and vertical ranges of the gun.[3] The total fire envelope of the target-defense system is the overall boundary of the individual envelopes which can be approximated as a smooth close surface. Since the radar is ineffective below a certain height, so are the radar-controlled guns.

The usual stragety adopted by the pilots in this mission is to dive towards the target after acquisition by piercing the fire envelope at some entry point, maneuver the aircraft so as to achieve a predetermined weapon release condition (termed pickle point), release the weapon and then fly out of the fire envelope at some exit point. In practice, weapon impact errors arise due to differences in the planned and actual weapon

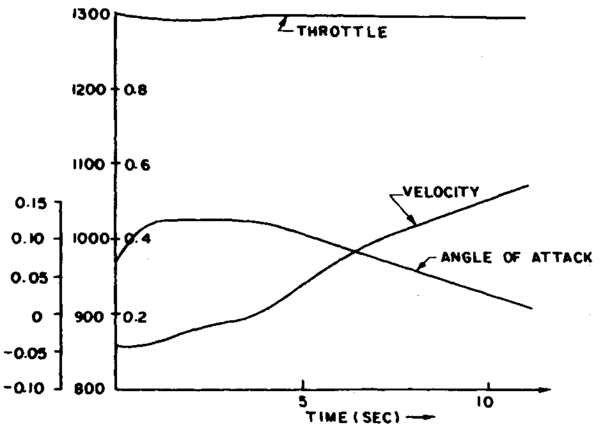

Fig. 1a Velocity and control variable programs (Case 1).

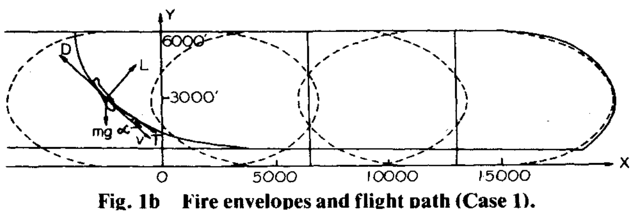

Fig. 1b Fire envelopes and flight path (Case 1).

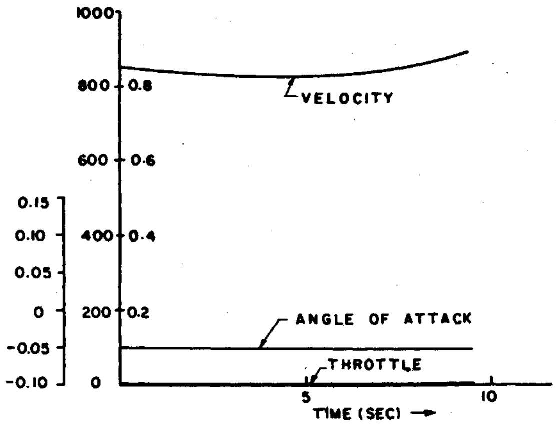

Fig. 2a Velocity and control variable programs (Case 2).

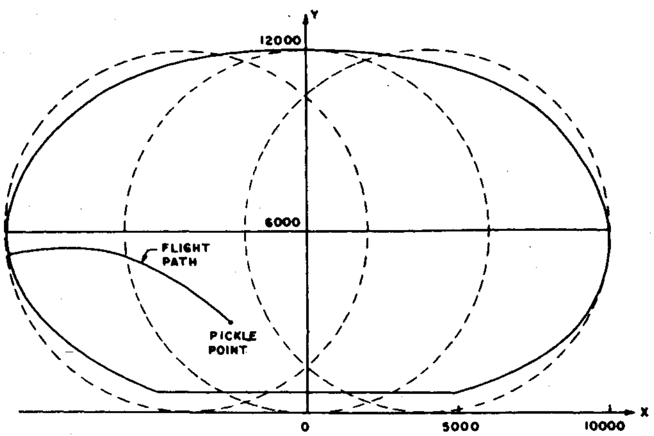

Fig. 2b Fire envelopes and flight path (Case 2).

release conditons, gust effects and uncertainty in the flight characteristics of the bomb.[1] Thus accuracy of bombing depends largely on the pickle point parameters and the handling qualities of the aircraft. Safety in the mission, on the other hand, depends only on the time of flight within the fire envelope.

Once the pickle point parameters are chosen so that the released weapon nominally falls on the target, on need only find the minimum-time pre- and post-pickle point trajectories for optimizing the mission effectiveness. Considering flight in a vertical plane through the target (see Fig. 1), the aircraft velocity, flight path angle, and position coordinates satisfy the state equations

$$\dot{v} = [T(v,y)\pi\cos\alpha - D(v,y,\alpha)]/m - g\sin\gamma \qquad (1)$$

$$\dot{\gamma} = [L(v,y,\alpha) + T(v,y)\pi\sin\alpha]/mv - (g/v)\cos\gamma \qquad (2)$$

$$\dot{x} = v\cos\gamma \qquad (3)$$

$$\dot{y} = v\sin\gamma \qquad (4)$$

where *T, L, D* are the trust, lift and drag functions of the aircraft. The control variables (angle of attack and throttle input) are constrained by $\alpha_{min} \leq \alpha \leq \alpha_{max}$ and $0 \leq \pi \leq 1$. Further the entry and exit points must lie on the fire envelope. If the exit point lies on the bottom flat portion of the fire envelope, the flight path angle should be zero so that the aircraft does not hit the ground. This extra constraint is actually met by a penalty function method.

The Hamiltonian and adjoint equations for both the pre- and post- pickle point problems are given by

$$H = 1 + \lambda_v \dot{v} + \lambda_\gamma \dot{\gamma} + \lambda_x \dot{x} + \lambda_y \dot{y} \qquad (5)$$

$$\dot{\lambda}_\alpha = \partial H / \partial \alpha; \; \alpha = v, \gamma, x, y \qquad (6)$$

Application of transversality conditions completely determines the adjoint variables at both the entry and exit points.[4] The first-order conjugate gradient method is chosen to obtain the optimal trajectories since it has minimal storage and programming requirements, accepts a poor initial guess and exhibits near-quadratic convergence.[5] For the pre-pickle point trajectory, the directions of integration of the state and adjoint equations get reversed for obvious reasons.

Results and Discussion

The aircraft used to test the software developed is an early version of F-4. The minimum and maximum angles of attack are assumed as −0.05 and 0.15 rad, respectively. The aerodynamic and thrust data are taken from Ref. 6.

The pickle point coordinates are assumed as (−2500 ft, 3000 ft). Taking the aircraft velocity at the pickle point as 900 fps, the corresponding flight path angle is computed for a low-drag-type bomb as 45°. Results for only two different fire envelopes are presented below for the sake of brevity.

Case 1

Two concentric circles of gun layout with radii 6,500 ft and 13,000 ft along with a gun very close to the target is assumed. The individual fire envelope of each gun is an ellipsoid with semi-axes as 7,000 ft and 3,000 ft. The optimal trajectory as well as the corresponding control programs of α and π are shown in Fig. 1. The post-pickle point trajectory takes 7.2 sec for the aircraft to reach a horizontal flight condition at 500 ft. This solution is very close to that conventionally flown.

Case 2

Here a single circle layout of guns with radius 4,000 ft along with a central gun each with spherical fire envelope of 6,000 ft is assumed. Results are shown in Fig. 2. The pre-pickle point trajectory is a glide path at the min. angle of attack indicating a fast sinking rate. The post-pickle point trajectory coincides with that for Case 1 and is not repeated in Fig. 2.

The best tradeoff between safety and accuracy can be arrived at through a subjective comparison of the results for different pickle points. A parameter optimization procedure iterating on the pickle point parameters is currently under development for automating this comparison.

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